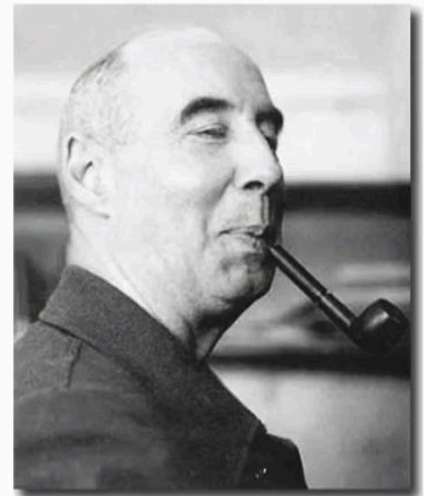


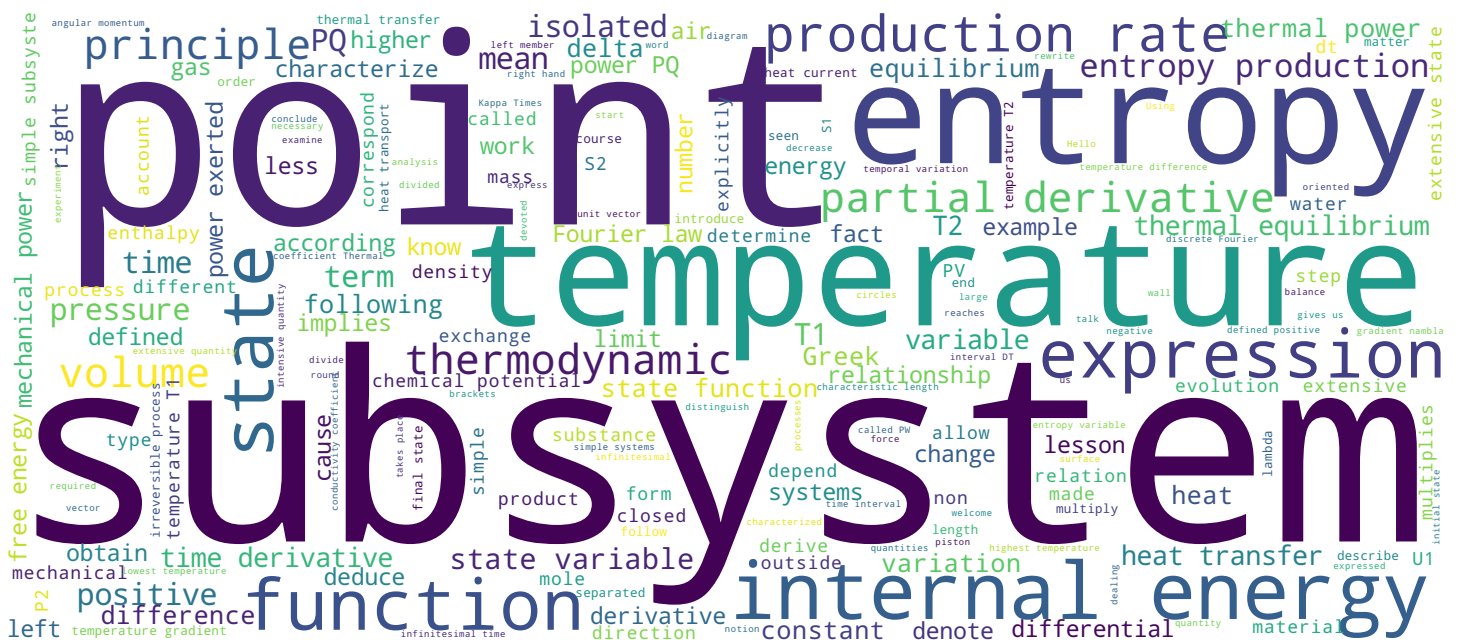
Thermodynamique

Equilibre thermique et transfert thermique

Dr. Sylvain Bréchet



Ernst Carl Gerlach Stükelberg, 1905 - 1984



Video





- Deux sous-systèmes simples séparés par une paroi diatherme, immobile et imperméable
- Equilibre thermique
- Transfert thermique (transport de chaleur)
- Loi de Fourier
 - Formulation discrète (2 sous-systèmes)
 - Formulation continue (∞ sous-systèmes)

Thermodynamique

Hello and welcome to this lack of thermodynamics. This lesson is devoted to thermal equilibrium and heat transport. We will consider an isolated system made up of two simple subsystems which are separated by a very dull, immobile and impermeable wall. In a first step, we will deduce the equilibrium condition and in a second step, we will examine the thermal transfer. We also talk about heat transport and we will then deduce the Fourier law. First, in its discrete formulation, i.e. for two subsystems, and then we will go to the limit of the continuum, i.e. when there is an infinity of subsystems.

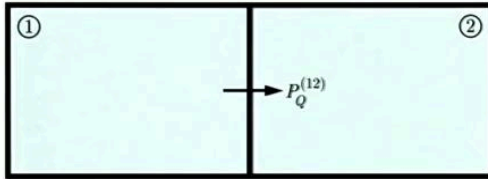
Notes

Summary



0m 05s

Paroi diatherme immobile et imperméable



- Système isolé : $P_Q = P_W = 0$
- Sous-systèmes simples (1) et (2)
- Paroi immobile : $P_W^{(12)} = P_W^{(21)} = 0$
- Variables d'état extensives :
 - Entropies S_1 et S_2

- Premier principe (sous-systèmes 1 et 2) :

$$\dot{U}_1(S_1) = T_1(S_1) \dot{S}_1 = P_Q^{(21)}$$

$$\dot{U}_2(S_2) = T_2(S_2) \dot{S}_2 = P_Q^{(12)}$$

- Energie interne (fonction d'état extensive) :

$$U(S_1, S_2) = U_1(S_1) + U_2(S_2)$$

- Premier principe (système isolé) :

$$\dot{U}(S_1, S_2) = \dot{U}_1(S_1) + \dot{U}_2(S_2) = P_Q^{(21)} + P_Q^{(12)} = 0$$

- Identités :

$$\dot{U}_1(S_1) = -\dot{U}_2(S_2) \quad \text{et} \quad P_Q^{(12)} = -P_Q^{(21)}$$

Thermodynamique

We consider an isolated system made up of two simple subsystems subsystem one on the left and sub-system two on the right. Since the system is isolated, the thermal power P_Q and the mechanical power P_W exerted by the outside on the system are zero. The two simple subsystems are separated by a stationary wall. This implies that the mechanical power exerted by the subsystem one on the subsystem two is the same. system two, which is called P_W one two is zero and analogously, the mechanical power exerted by subsystem two on subsystem one, which is called P_W of one is also zero. To characterize the thermodynamics of this system, an extensive type of state variable is required. This is entropy. We have two sub-systems which are simple systems. By definition, a simple system is characterized by a single entropy variable. It takes two entropy variables an entropy S_1 which characterizes the subsystem one and one between S_2 which characterizes the subsystem of. Internal energy and temperature are state functions. So the internal energy and temperature of a subsystem will be functions of the variables of state of this subsystem, in this case of the subsystem entropy U .

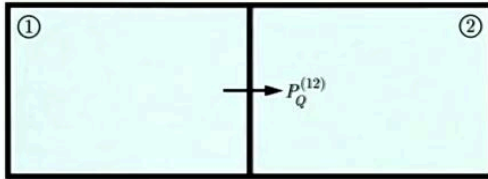
Notes

Summary



0m 55s

Paroi diatherme immobile et imperméable



- **Système isolé :** $P_Q = P_W = 0$
- **Sous-systèmes simples (1) et (2)**
- **Paroi immobile :** $P_W^{(12)} = P_W^{(21)} = 0$
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$$\dot{U}_1(S_1) = T_1(S_1) \dot{S}_1 = P_Q^{(21)}$$

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- **Energie interne (fonction d'état extensive) :**

$$U(S_1, S_2) = U_1(S_1) + U_2(S_2)$$
- **Premier principe (système isolé) :**

$$\dot{U}(S_1, S_2) = \dot{U}_1(S_1) + \dot{U}_2(S_2) = P_Q^{(21)} + P_Q^{(12)} = 0$$
- **Identités :**

$$\dot{U}_1(S_1) = -\dot{U}_2(S_2) \quad \text{et} \quad P_Q^{(12)} = -P_Q^{(21)}$$

Thermodynamique

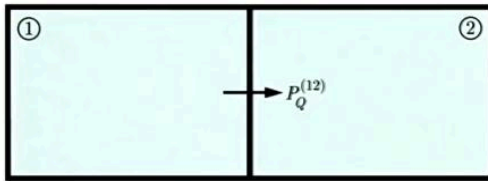
One point is T1 is simple and according to the first principle of thermodynamics, we know that the cause of the temporal variation of the internal energy of the first sub system, it is the thermal power exerted by the second subsystem. On the first subsystem, it is PQ. Two one. In a similar way. More points is T2. Is it two points? According to the first principle, we know that the cause which causes the temporal variation of the internal energy of the second sub system, it is the thermal power exerted by the first subsystem on the second subsystem which is noted PQ one. Two. the internal energy is a state function. It is therefore a function of the set state variables of the system, i.e. state variables of the first sub-system and the second sub-system. Therefore, U is a function of S1 and also of S2. It is an extensive state function. This means that the internal energy of the system U is the sum of the internal energy of the first subsystem one and the internal energy of the second subsystem U two. We can take the time derivative of this expression. So we have that one point is equal to one point plus two points. Given the first principle, we can rewrite a point. As the PQ must.

Notes

Summary



Paroi diatherme immobile et imperméable



- **Système isolé :** $P_Q = P_W = 0$
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- **Paroi immobile :** $P_W^{(12)} = P_W^{(21)} = 0$
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- **Energie interne (fonction d'état extensive) :**

$$U(S_1, S_2) = U_1(S_1) + U_2(S_2)$$
- **Premier principe (système isolé) :**

$$\dot{U}(S_1, S_2) = \dot{U}_1(S_1) + \dot{U}_2(S_2) = P_Q^{(21)} + P_Q^{(12)} = 0$$
- **Identités :**

$$\dot{U}_1(S_1) = -\dot{U}_2(S_2) \quad \text{et} \quad P_Q^{(12)} = -P_Q^{(21)}$$

Thermodynamique

And we can rewrite two points like PQ into two. In addition, the system is isolated. This means that according to the first In principle, the time derivative of the internal energy of the point is zero. This gives us two identities. First, one point is equal to fewer points. This means that if the internal energy in both subsystems increases, the internal energy of the other subsystem will decrease and vice versa. And the second relationship? The second identity is that PQ12 is equal to minus p q of one.

Notes

Summary



4m 21s



- Entropie (variable d'état extensive) :

$$S = S_1 + S_2$$

- Dérivée temporelle de l'entropie :

$$\dot{S} = \dot{S}_1 + \dot{S}_2 = \frac{\dot{U}_1(S_1)}{T_1(S_1)} + \frac{\dot{U}_2(S_2)}{T_2(S_2)}$$

- Système isolé : $\dot{U}_2(S_2) = -\dot{U}_1(S_1)$

$$\dot{S} = \left(\frac{1}{T_1(S_1)} - \frac{1}{T_2(S_2)} \right) \dot{U}_1(S_1)$$

- Identités : $dS = \dot{S} dt$ et $dU_1 = \dot{U}_1 dt$

- Dérivée partielle de l'entropie :

$$\frac{\partial S}{\partial U_1} = \frac{1}{T_1(S_1)} - \frac{1}{T_2(S_2)}$$

Thermodynamique

We will now take into account explicitly from the fact that entropy is an extensive state variable. This means that the entropy of the system S is the sum of the entropies of the two sub-systems. systems, i.e. S is equal to S1 plus S2. We now take the time derivative of the entropy. Is this point? It's a plus point. Is it two points? And we know that one point is equal to T1 and five points. So S 1.7 U a point on T1. Err. Two points is two times. Is it two points? So is it 2.7 points out of two? We now explicitly take into account the fact that the system is isolated. As we have seen, if the system is isolated, more points is equal to less than one point. Therefore. Is this the point? It is equal to S one point more. S 2.71 on T1 -1 on T2 which multiplies u a point. We will now reformulate this relationship to express it explicitly in terms of a differential. For this, we will multiply the left members and on the right by the infinitesimal time interval DT. And we will obtain in the left member the differential DS and in the right-hand member the differential d u. This now allows us to take the partial derivative of the entropy with respect to U1.

Notes

Summary



5m 05s



- Dérivée partielle de l'entropie :

$$\frac{\partial S}{\partial U_1} = \frac{1}{T_1(S_1)} - \frac{1}{T_2(S_2)}$$

- Deuxième principe (condition d'équilibre) :

$$\frac{\partial S}{\partial U_1} = 0 \quad (\text{maximum d'entropie})$$

- Equilibre thermique :

$$T_1(S_1) = T_2(S_2)$$

Le premier et le deuxième principes requièrent que les températures des sous-systèmes aient la même valeur à l'équilibre thermique.

Thermodynamique

So rounds s on rounds u1 is equal to one on T1 -1 on T2 in order to determine the thermal equilibrium condition. We will now use the equilibrium condition of the second principle which states that for an isolated system, entropy is maximal at equilibrium. This implies that the partial derivative of the entropy S with respect to U1 is zero. Given the expression of the partial derivative of S with respect to U1, we conclude that the two fractions of the right hand side must be equal, which gives us the condition of thermal equilibrium. This thermal equilibrium condition states that the temperature T1 of the first The temperature of the first sub-system must be equal to the temperature T2 of the second sub-system. Therefore, the first and second principles of thermodynamics require that the temperatures of the subsystems and the same value at thermal equilibrium before the system reaches a state of thermal equilibrium.

Notes

Summary



6m 51s

- Dérivée temporelle de l'entropie ($T_1 \neq T_2$) :

$$\dot{S} = \left(\frac{1}{T_1(S_1)} - \frac{1}{T_2(S_2)} \right) \dot{U}_1(S_1)$$

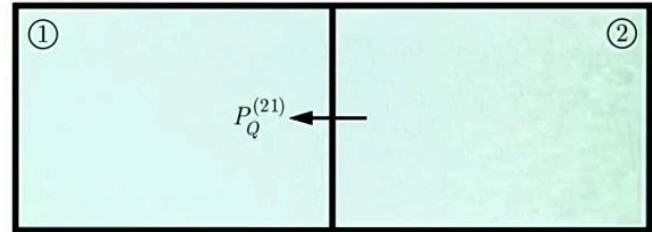
- Système isolé (processus irréversible) :

$$\dot{S} = \Pi_S > 0 \quad \text{et} \quad \dot{U}_1(S_1) = P_Q^{(21)}$$

- Taux de production d'entropie :

$$\Pi_S = \left(\frac{1}{T_1(S_1)} - \frac{1}{T_2(S_2)} \right) P_Q^{(21)} > 0$$

- $T_2 > T_1 \Rightarrow P_Q^{(21)} > 0$
- $T_1 > T_2 \Rightarrow P_Q^{(12)} = -P_Q^{(21)} > 0$



- Transfert thermique (transport de chaleur) :

- Chaleur : $T_+ \Rightarrow T_-$
- Processus irréversible : $\Pi_S > 0$
- Nul à l'équilibre thermique : $T_1 = T_2$

- Taux de production d'entropie :

$$\Pi_{S_1} = \Pi_{S_2} = 0 \Rightarrow \Pi_S \neq \Pi_{S_1} + \Pi_{S_2}$$

Thermodynamique

There is a heat transfer that takes place between these two subsystems. So, before he reaches this state of thermal equilibrium, the temperatures of the two subsystems are different and therefore T_1 is not equal to T_2 . To examine this heat transfer. We will base our analysis on the expression of the time derivative of entropy. This expression is the following. Is this the point? Is equal to one on T_1 one on two times one point. Like T_1 and T_2 . The difference of the terms here in brackets is non-zero. As there is heat transfer. the internal energy of the first subsystem. Varies therefore a point and non-zero. This implies that this point is non-zero. We are dealing with an isolated system. For an isolated system. According to the second principle is this point is equal to the entropy production rate πds . And as a hope and not a draw. It means by the second principle that it is positive. We are therefore dealing with an irreversible process. Furthermore, the first principle states that one point is equal to P_Q two one. As a consequence of the expression of the time derivative of entropy, we derive an expression for the entropy production rate.

Notes

Summary



- Dérivée temporelle de l'entropie ($T_1 \neq T_2$) :

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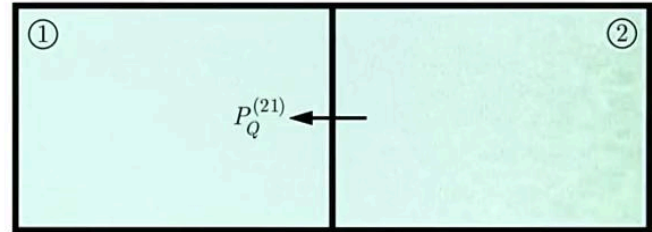
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- Taux de production d'entropie :

$$\Pi_{S_1} = \Pi_{S_2} = 0 \Rightarrow \Pi_S \neq \Pi_{S_1} + \Pi_{S_2}$$

Thermodynamique

The expression is the following Hessian Pie is equal to one on T1 month one out of two times PQ of and this entropy production rate is positive. We must now distinguish two cases. In the first case, the temperature T2 of the second subsystem. Is higher than the temperature T1. From the first subsystem. Therefore, the difference in the terms in parentheses in the expression of the entropy production rate, this difference is positive. So that the entropy production rate is positive. This implies that p q of one is positive. Therefore, when the temperature of the second subsystem is higher than the temperature of the first subsystem, there is a heat transfer that will from the second subsystem to the first subsystem. Let us now consider the second case. Second case, the temperature T1 of the first subsystem is higher than the temperature T2 of the second subsystem. Which means. That the difference in terms here in brackets in the expression of the entropy production rate, this difference is negative. Therefore, for the entropy production rate is defined positive, it is necessary that p q of one is also negative. PQ two one is equal to minus PQ one two, which means that PQ one two is positive.

Notes

Summary



- Dérivée temporelle de l'entropie ($T_1 \neq T_2$) :

$$\dot{S} = \left(\frac{1}{T_1(S_1)} - \frac{1}{T_2(S_2)} \right) \dot{U}_1(S_1)$$

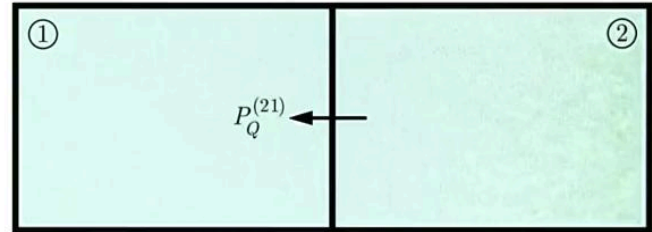
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- Taux de production d'entropie :

$$\Pi_{S_1} = \Pi_{S_2} = 0 \Rightarrow \Pi_S \neq \Pi_{S_1} + \Pi_{S_2}$$

Thermodynamique

Therefore, if the temperature of the first subsystem T_1 is higher than the temperature of the second sub-system T_2 , there will be a heat transfer that will from the first subsystem to the second subsystem. In summary, heat transfer or heat transport goes to the subsystem with the highest temperature. Let us note the T_+ . To the subsystem with the lowest temperature. Note we witnessed it. This thermal transfer is an irreversible process, i. e. copy of Hess and positive. The thermal transfer allows the system to to reach a state of thermal equilibrium, that is to say that the system will tend towards a state of thermal equilibrium. The temperatures of the two subsystems T_1 is equal to T_2 . These two temperatures are equal. And therefore in this limit, the heat transfer is zero. We saw in a previous lesson that for a simple, rigid system, closed, with walls said to have a zero entropy production rate. This is the case. Of the two subsystems. These two subsystems are simple systems. So no breasts is equal to S_2 and they are zero. Clearly the production rate of entropy of the whole Hessian system and not zero. It is positive, which means that Π_S is not equal to $\Pi_{S_1} + \Pi_{S_2}$. We therefore conclude that the entropy production rate is not an extensive quantity and it is not an intensive quantity either.

Notes

Summary



11m 35s



- Taux de production d'entropie :

$$\Pi_S = \left(\frac{T_2(S_2) - T_1(S_1)}{T_1(S_1) T_2(S_2)} \right) P_Q^{(21)} > 0$$

- Loi de Fourier « discrète » :

$$P_Q^{(21)} = \kappa \frac{A}{\ell} (T_2(S_2) - T_1(S_1))$$

κ : Conductivité thermique

A : Aire de l'interface entre (1) et (2)

ℓ : Longueur caractéristique

Thermodynamique

We can now deduce from this analyses the Fourier law in its discrete formulation. First, we can re-shape the production rate of entropy which is expressed as follows Π_S is equal to $T_2 - T_1$ on $T_1 T_2$. The however P_Q of one and this entropy production rate is defined positive. In order to guarantee that this entropy production rate is defined as positive. The difference between the temperatures in the expression of the entropy production rate appears squared. In other words, P_Q of one must be proportional to $T_2 - T_1$. A proportionality factor that is positive is required. This is precisely the discrete Fourier law. Thermal power exerted by the second subsystem on the first subsystem P_Q two one is equal to the product of the conductivity coefficient Thermal that we denote by the letter κ . Times the area of the interface between the two subsystems which is denoted large A divided by a characteristic length that we denote ℓ and the temperature difference between the two subsystems, i.e. P_2 minus.

Notes

Summary



Loi de Fourier "continue"

- Loi de Fourier « discrète » :

$$P_Q = \kappa \frac{A}{\ell} (T_+ - T_-)$$

- Gradient de température :

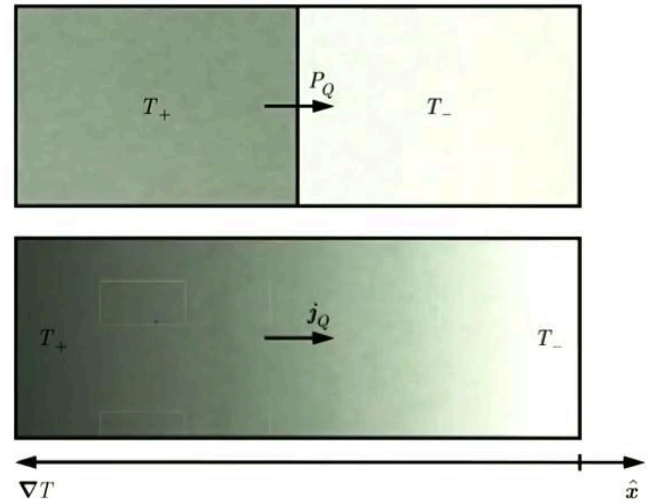
$$\nabla T = - \frac{T_+ - T_-}{\ell} \hat{x}$$

- Densité de courant de chaleur :

$$j_Q = \frac{P_Q}{A} \hat{x}$$

- Loi de Fourier « continue » :

$$j_Q = -\kappa \nabla T$$



Thermodynamique

T1. In physics, we are often dealing with systems in which the temperature varies gradually and linearly between one end of the system and another end of the system. Let's take a metal bar for example. Suppose that the left end is the hot end with the highest temperature T_+ , and that the right end is the cold end in the lowest temperature control. To be able to describe the Fourier law in a continuous way. We cannot limit ourselves to two subsystems with fixed temperatures. We have to consider an infinite number of subsystems whose length is an infinitesimal length. So we have a temperature variation when going from the right to the left, this variation is linear. To account for this variation, we introduce the so-called temperature gradient ∇T . This gradient corresponds to a variation of temperature in the direction of increasing temperature. The \hat{x} vector is here a vector unit which is oriented to the right and the gradient temperature is oriented to the left. Therefore, since the length of the system corresponds to the characteristic length L , the temperature gradient is noted as follows. Note Blatter. It's less and more of a witness the temperature difference between the ends divided by L .

Notes

Summary



15m 06s

Loi de Fourier “continue”

- Loi de Fourier « discrète » :

$$P_Q = \kappa \frac{A}{\ell} (T_+ - T_-)$$

- Gradient de température :

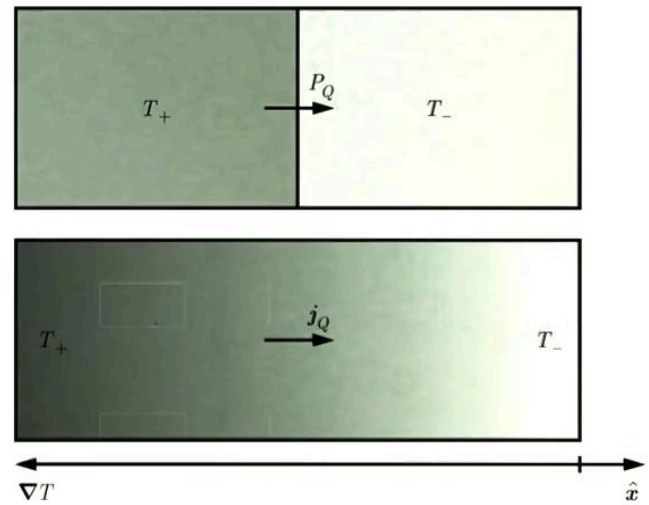
$$\nabla T = - \frac{T_+ - T_-}{\ell} \hat{x}$$

- Densité de courant de chaleur :

$$j_Q = \frac{P_Q}{A} \hat{x}$$

- Loi de Fourier « continue » :

$$j_Q = -\kappa \nabla T$$



Thermodynamique

The length of the system. \hat{x} times hat. The unit vector in the direction of heat transfer. We must now introduce another physical quantity which is the density of heat current that we will denote by a large j index q . This heat current density is defined as follows. It is the ratio of the thermal power P_Q on area A which corresponds to the area that is orthogonal to the heat transfer. And this heat transfer takes place in the direction of the unit vector \hat{x} which has no physical dimension. Therefore, if we take the discrete Fourier law here, divide it by large A and multiply it by \hat{x} . Hats off to you! We derive the Fourier law in its continuous formulation, which is the following. The density of the heat current on the left is equal to minus the conductivity coefficient thermal κ times the temperature gradient ∇T .

Notes

Summary



16m 49s